# MAT 303 Module Three Problem Set Report

Second Order Models

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## **1. Introduction**

In this analysis, we are exploring a data set economic containing historical data used to study wage growth, The primary objective is to understand how changes in these economic predictors impact wage growth.

We will use descriptive statistics to summarize the dataset and provide an overview. Scatterplot analysis will help us visually explore the relationships between wage growth and unemployment. We'll employ a second-order regression model to capture the nonlinear relationships among wage growth, unemployment, GDP growth, and economy incorporating both linear and squared predictor terms. Model evaluation will utilize R-squared and Adjusted R-squared to assess variability in wage growth, along with F-tests and T-tests for overall model significance and individual predictors.

By conducting these analyses, we aim to gain a deeper understanding of the dynamics between wage growth, unemployment, and GDP growth, providing valuable insights for economic decision-making.

The results of this analysis can be used by policymakers, economists, and business leaders to make informed decisions. For instance, understanding the relationship between unemployment, GDP growth, and wage growth can help in formulating economic policies, setting wage standards, and predicting future economic trends. Businesses can use this information to plan their hiring strategies and wage adjustments based on expected changes in economy key factors unemployment and GDP growth rates.

## **2. Data Preparation**

The data set “economic” contains 5 columns of variables and 99 rows of data.

* **Wage growth rate:** The rate at which wages are increasing.
* **Inflation rate:** The rate of inflation.
* **Unemployment rate:** The percentage of the labor force that is unemployed.
* **Economy:** Indicates whether the economy is in recession or not.
* **GDP growth rate:** The rate at which the Gross Domestic Product is growing.

There are 3 Quadratic (Second Order) models focused on around the variable Wage Growth (wage\_growth) rate as follows:

1. Second Order model1 with one quantitative Variable Unemployment Rate (unemployment)
2. Second Order regression model2 with two quantitative variables Unemployment Rate (unemployment) and GDP growth rate (GDP)
3. Second Order regression model3 with one quantitative Unemployment Rate (unemployment) and one qualitative variable Economy (economy).

## **3. Quadratic (Second Order) Model with One Quantitative Variable**

### Correlation Analysis

**A graph with purple dots

Description automatically generated**

The scatterplot of wage growth and unemployment shows a clear nonlinear relationship. As unemployment increases, wage growth initially decreases rapidly, but the rate of decrease slows down at higher levels of unemployment. This pattern suggests a parabolic relationship between the two variables.

Given this observed curvature, a first order (linear) model would not adequately capture the relationship. Instead, a second order (quadratic) model is more appropriate because it can account for the changing rate of wage growth as unemployment increases. The quadratic model allows for the initial rapid decline and the subsequent leveling off, providing a better fit for the data.

### Reporting Results

For wage growth using unemployment as the predictor variable, the general form of the second-order regression model is:

The prediction equation based on the output:

**Interpretation R-squared and Adjusted R-squared Values**

The R-squared value of 0.9436 indicates that approximately 94.36% of the variability in wage growth is explained by the model. This demonstrates a strong fit, meaning the model captures a significant portion of the variation in wage growth. The Adjusted R-squared value of 0.9424 adjusts for the number of predictors in the model and still shows a high explanatory power at 94.24%. This confirms that the model is robust and not overly complex.

***Interpretation of Beta Estimates***

Regarding beta estimates, the intercept is 12.2342, which represents the estimated wage growth when unemployment is zero. The coefficient for unemployment is -1.7432. This negative value indicates that as unemployment increases, wage growth initially decreases. The coefficient for unemployment squared is 0.0674. This positive value suggests that the rate of decrease in wage growth slows down at higher levels of unemployment, forming a parabolic relationship.

### Evaluating Model Significance

To determine if the second-order regression model for wage growth using unemployment as the predictor variable is significant at a 5% level of significance, we perform an overall F-test. The null hypothesisstates that the model with no predictors fits the data as well as the current model, implying that all coefficients except the intercept are equal to zero . The alternative hypothesis suggests that at least one of the predictors is significantly related to the response variableThe p-value for the F-test is reported as less than , which is much smaller than the 0.05 significance level. Therefore, we reject the null hypothesis, concluding that the model is significant at the 5% level of significance. This indicates that at least one of the predictors (unemployment or unemployment squared) is significantly related to wage growth.

Next, we examine the significance of individual terms in the model using T-tests at a 5% level of significance. The p-value for the intercept is less thanmaking it significant. Similarly, the p-values for the unemployment term (0.2869) and the unemployment squared term (-0.0146) are both less than *,* indicating that these terms are also significant. Thus, all terms in the model are significant at the 5% level of significance. This means that both the linear and quadratic components of unemployment are important predictors of wage growth, with the positive coefficient for unemployment suggesting an initial increase in wage growth as unemployment rises, and the negative coefficient for unemployment squared indicating that this positive relationship diminishes at higher levels of unemployment, forming a parabolic relationship.

### Making Predictions Using Model

Using the regression model, the predicted wage growth when unemployment is 2.54 can be calculated using the prediction equation

So, the predicted wage growth is approximately 8.2423.

The 95% prediction interval for wage growth is given as 8.0071 to 9.5758. This interval suggests that there is a 95% chance that the actual wage growth for an unemployment rate of 2.54 will fall within this range. It accounts for the variability in individual predictions.

The 95% confidence interval for wage growth is given as 8.0936 to 8.3893. This interval indicates that we can be 95% confident that the average wage growth for an unemployment rate of 2.54 falls within this range. It provides an estimate of the mean response.

## **4. Complete Second Order Model with Two Quantitative Variables**

### Reporting Results

`The general form of a complete second-order regression model for wage growth as the response variable, with unemployment and GDP growth as predictor variables, is:

*Prediction Model Equation based on the outputs for wage growth is:*

***Interpretation R-squared and Adjusted R-squared Values***

The R-squared value of 0.9587 indicates that approximately 95.87% of the variability in wage growth is explained by the model. This demonstrates a very strong fit, meaning the model captures a significant portion of the variation in wage growth. The Adjusted R-squared value of 0.9565 adjusts for the number of predictors in the model and still shows a high explanatory power at 95.65%. This confirms that the model is robust and not overly complex.

***Interpretation of Beta Estimates***

The beta estimates, the coefficient for GDP squared is -0.0066. This negative value suggests that as GDP growth increases, the rate of increase in wage growth slows down, indicating a diminishing return effect. The coefficient for unemployment squared is 0.0377. This positive value indicates that as unemployment increases, the negative impact on wage growth diminishes, forming a parabolic relationship. Overall, the model indicates a complex relationship between wage growth, unemployment, and GDP growth, with both linear and quadratic terms contributing to the prediction.

### Evaluating Model Significance

**Overall F-test for Model Significance**

To evaluate the model significance for the regression model, we start with the overall F-test. The null hypothesis states that the model with no predictors fits the data as well as the current model, meaning all coefficients except the intercept are equal to zero . The alternative hypothesis posits that at least one of the predictors is significantly related to the response variable *.* The p-value for the F-test is reported as . Since this p-value is much smaller than 0.05, we reject the null hypothesis, concluding that the model is significant at the 5% level of significance. This indicates that at least one of the predictors is significantly related to wage growth.

Now let’s examine the significance of individual terms in the model using T-tests at a 5% level of significance. The intercept has an estimate of 8.9894 with a p-value of , making it significant. The unemployment term has an estimate of -1.1528 with a p-value of 8.26e-06, also significant. The GDP term has an estimate of 0.2837 with a p-value of 0.04682, which is significant as well. The unemployment squared term has an estimate of 0.0377 with a p-value of 0.00489, indicating significance. However, the GDP squared term has an estimate of -0.0066 with a p-value of 0.12815, which is not significant. Similarly, the interaction term between unemployment and GDP has an estimate of -0.0063 with a p-value of 0.76678, also not significant.

Concluding that the overall model is significant at the 5% level of significance. The individual terms for the intercept, unemployment, GDP, and unemployment squared are significant, while the terms for GDP squared and the interaction between unemployment and GDP are not significant.

### Making Predictions Using Model

According to the regression model, the predicted wage growth when unemployment is at 2.50% and GDP growth is at 6.50% is estimated to be 7.806%.

The 95% prediction interval for wage growth is [6.6315, 8.9805]. This implies that we can be 95% confident that the actual wage growth for a new observation under these conditions will fall within this range.

Additionally, the 95% confidence interval for wage growth is [7.583, 8.0289]. This interval indicates that we can be 95% confident that the true mean wage growth for these conditions lies within this range.

## **5. Complete Second Order Model with One Quantitative and One Qualitative Variable**

### Reporting Results

The general form of a complete second-order regression model for wage growth using unemployment and economy as predictor variables is:

*Or*

*Based on your R output, the prediction model equation is*

The R-squared value for the model is 0.9475, indicating that approximately 94.75% of the variability in wage growth can be explained by the model. This demonstrates a strong relationship between the predictors (unemployment and economy) and the response variable (wage growth). The Adjusted R-squared value is 0.9446. This statistic adjusts for the number of predictors in the model, suggesting that the model is very effective in explaining the variability in wage growth, even after accounting for the number of predictors.

### Evaluating Model Significance

To evaluate the significance of the regression model at a 5% level of significance, we can perform an overall F-test. The null hypothesisfor the F-test states that all regression coefficients are equal to zero, meaning the model has no explanatory power. The alternative hypothesisstates that at least one regression coefficient is not equal to zero, indicating that the model is significant. Based on the output, the P-value for *the* F-test *,* which is much smaller than the *0.05* significance level. Therefore, we reject the null hypothesis and conclude that the model is significant at the 5% level.

Next, we evaluate the significance of individual terms in the model using T-tests at a 5% level of significance. The null hypothesis for each T-test states that the corresponding regression coefficient is equal to zero. The P-values for the coefficients of unemployment, economy, , and the interaction term *(unemployment*  are all less than 0.05, indicating that these terms are significant. However, the P-value for the coefficient of is 0.0512, which is slightly above the 0.05 threshold, suggesting that this term is not significant at the 5% level.

So, the overall model is significant at the 5% level, and the significant terms in the model are unemployment, economy*, ,* and the interaction term *(unemployment .*

### Making Predictions Using Model

Using the model and setting unemployment at 2.50 and indicating that the economy is ‘not in recession,’ the predicted wage growth is 8.3132.

The 95% prediction interval for wage growth is [7.0003, 9.6251]. This implies that we can be 95% confident that the actual wage growth will fall within this range for a new observation under the specified conditions.

Conversely, the 95% confidence interval for wage growth is [8.1573, 8.4692]. This interval suggests that we can be 95% confident that the true mean wage growth under the given conditions lies within this range.

The prediction interval is wider than the confidence interval because it takes into account both the variability in the estimated mean response and the variability of individual observations around the mean. In contrast, the confidence interval only addresses the variability associated with estimating the *mean response.*

## **6. Conclusion**

Based on the analysis performed and assuming that the sample size is sufficiently large, I would recommend using this model. The high R-squared and adjusted R-squared values indicate that the model explains a large proportion of the variability in wage growth. Additionally, the significant coefficients for unemployment, economy, and their interactions suggest that these predictors are important for understanding wage growth.

The results indicate that both unemployment and the state of the economy (recession or no recession) significantly influence wage growth. The interaction terms highlight that the relationship between unemployment and wage growth varies depending on whether the economy is in recession. The quadratic terms suggest a nonlinear relationship, where the effect of unemployment on wage growth changes at different levels of unemployment.

The practical importance of these analyses is significant for policymakers and economists. Understanding the factors that influence wage growth can help in designing effective economic policies and interventions. For example, policies aimed at reducing unemployment could have a positive impact on wage growth, especially in times of economic stability. Additionally, the model can be used for forecasting wage growth under different economic scenarios, providing valuable insights for decision-making in both the public and private sectors.